

## Relational Algebra

- Relational algebra is the basic set of operations for the relational model
- These operations enable a user to specify **basic retrieval requests** (or **queries**)

## Relational Algebra Overview

- Relational Algebra consists of several groups of operations
  - Unary Relational Operations
    - SELECT (symbol:  $\sigma$  (sigma))
    - PROJECT (symbol:  $\pi$  (pi))
    - RENAME (symbol:  $\rho$  (rho))

## Relational Algebra Overview

- Relational Algebra Operations From Set Theory
  - UNION ( $\cup$ ),
  - INTERSECTION ( $\cap$ ),
  - DIFFERENCE (or MINUS,  $-$ )
  - CARTESIAN PRODUCT ( $\times$ )

## Relational Algebra Overview

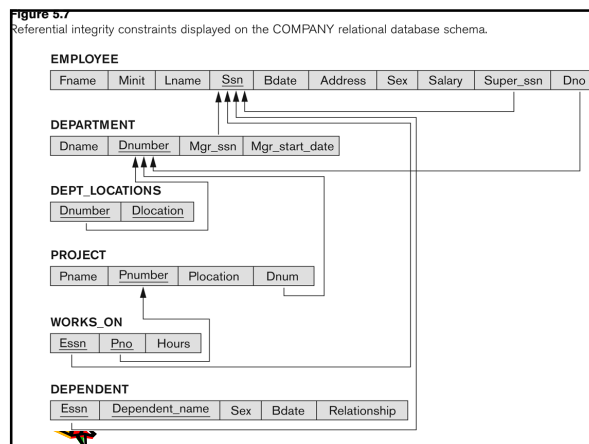
- Binary Relational Operations
  - JOIN (several variations of JOIN exist)
  - DIVISION
- Additional Relational Operations
  - OUTER JOINS, OUTER UNION
  - AGGREGATE FUNCTIONS

## Unary Relational Operations

- SELECT (symbol:  $\sigma$  (sigma))
- PROJECT (symbol:  $\pi$  (pi))
- RENAME (symbol:  $\rho$  (rho))

## Database State for COMPAN

- All examples discussed below refer to the COMPANY database shown here.



## SELECT

- The SELECT operation (denoted by  $\sigma$  (sigma)) is used to select a *subset* of the tuples from a relation based on a selection condition
  - The selection condition acts as a **filter**
  - Keeps only those tuples that satisfy the qualifying condition
  - Tuples satisfying the condition are *selected* whereas the other tuples are discarded (*filtered out*)

## SELECT

- Examples:
  - Select the EMPLOYEE tuples whose department number is 4:
 
$$\sigma_{DNO = 4} (EMPLOYEE)$$
  - Select the employee tuples whose salary is greater than K30,000:
 
$$\sigma_{SALARY > 30,000} (EMPLOYEE)$$

## SELECT

- In general, the *select* operation is denoted by  $\sigma_{\langle \text{selection condition} \rangle} (R)$  where
  - the symbol  $\sigma$  (sigma) is used to denote the *select* operator
  - the selection condition is a Boolean (conditional) expression specified on the attributes of relation R
  - tuples that make the condition **true** are selected
  - tuples that make the condition **false** are filtered

## SELECT

- The Boolean expression specified in  $\langle \text{selection condition} \rangle$  is made up of a number of clauses of the form:
  - $\langle \text{attribute name} \rangle \langle \text{comparison op} \rangle \langle \text{constant value} \rangle$
  - or
  - $\langle \text{attribute name} \rangle \langle \text{comparison op} \rangle \langle \text{attribute name} \rangle$
 Where  $\langle \text{attribute name} \rangle$  is the name of an attribute of R,  $\langle \text{comparison op} \rangle$  is normally one of the operations  $\{=, >, >=, <, <=, !=\}$   
 Clauses can be arbitrarily connected by the Boolean operators **and**, **or** and **not**

## SELECT

- For example, to select the tuples for all employees who either work in department 4 and make over K25,000 per year, or work in department 5 and make over K30,000, the select operation should be:

$\sigma_{(DNO=4 \text{ AND } Salary>25000) \text{ OR } (DNO=5 \text{ AND } Salary>30000)}(EMPLOYEE)$

## SELECT

- SELECT Operation Properties

- SELECT  $\sigma$  is commutative:

$$\sigma_{\langle \text{condition1} \rangle}(\sigma_{\langle \text{condition2} \rangle}(R)) = \sigma_{\langle \text{condition2} \rangle}(\sigma_{\langle \text{condition1} \rangle}(R))$$

- A cascade of SELECT operations may be replaced by a single selection with a conjunction of all the conditions:

$$\sigma_{\langle \text{cond1} \rangle}(\sigma_{\langle \text{cond2} \rangle}(\sigma_{\langle \text{cond3} \rangle}(R))) = \sigma_{\langle \text{cond1} \rangle \text{ AND } \langle \text{cond2} \rangle \text{ AND } \langle \text{cond3} \rangle}(R)$$

## PROJECT

- PROJECT Operation is denoted by  $\pi$  ( $\pi$ )
- If we are interested in only certain attributes of relation, we use PROJECT
- This operation keeps certain *columns* (attributes) from a relation and discards the other columns.

## PROJECT

- PROJECT creates a vertical partitioning
  - The list of specified columns (attributes) is kept in each tuple
  - The other attributes in each tuple are discarded

## PROJECT

- Example: To list each employee's first and last name and salary, the following is used:

$\pi_{LNAME, FNAME, SALARY}(EMPLOYEE)$

## Duplicate elimination

- If the attribute list includes only non-key attributes, duplicate tuples are likely to occur
- The PROJECT operation removes any duplicate tuples

### Single expression versus sequence of relational operations

- We may want to apply several relational algebra operations one after the other
- Either we can write the operations as a **single relational algebra expression** by nesting the operations, or



### Single expression versus sequence of relational operations

- We can apply one operation at a time and create **intermediate result relations**.

In the latter case, we must give names to the relations that hold the intermediate results.



### Single expression versus sequence of relational operations

- To retrieve the first name, last name, and salary of all employees who work in department number 5.
- We can write a *single relational algebra expression* as follows:

$$\pi_{\text{FNAME, LNAME, SALARY}}(\sigma_{\text{DNO}=5}(\text{EMPLOYEE}))$$


### Single expression versus sequence of relational operations

- OR We can explicitly show the *sequence of operations*, giving a name to each intermediate relation:
  - $\text{DEP5\_EMPS} \leftarrow \sigma_{\text{DNO}=5}(\text{EMPLOYEE})$
  - $\text{RESULT} \leftarrow \pi_{\text{FNAME, LNAME, SALARY}}(\text{DEP5\_EMPS})$



### Relational Algebra Operations from Set Theory

- Union
- Intersection
- Minus
- Cartesian Product



### UNION

- It is a Binary operation, denoted by  $\cup$
- The result of  $R \cup S$ , is a relation that includes all tuples that are either in R or in S or in both R and S
- Duplicate tuples are eliminated



## UNION

- The two operand relations R and S must be "type compatible" (or UNION compatible)
  - R and S must have same number of attributes
  - Each pair of corresponding attributes must be type compatible (have same or compatible domains)

## UNION

- Example:
  - To retrieve the social security numbers of all employees who either work in department 5 (RESULT1 below) or directly supervise an employee who works in department 5 (RESULT2 below)

## UNION

$DEP5\_EMPS \leftarrow \sigma_{DNO=5}(EMPLOYEE)$   
 $RESULT1 \leftarrow \pi_{SSN}(DEP5\_EMPS)$   
 $RESULT2 \leftarrow \pi_{SUPERSSN}(DEP5\_EMPS)$   
 $RESULT \leftarrow RESULT1 \cup RESULT2$

- The union operation produces the tuples that are in either RESULT1 or RESULT2 or both

## The following query results refer to this database state

Figure 6.3

EMPLOYEE	EMPLOYEE	EMPLOYEE	DEPT LOCATIONS	WORKS ON	PROJECT	DEPENDENT
Empno	Empno	Empno	Department	Empno	Project	Empno
John 123456789	John 123456789	John 123456789	Research	John 123456789	ProjectX	John 123456789
Franklin 333445555	Franklin 333445555	Franklin 333445555	Headquarters	Franklin 333445555	ProjectY	Franklin 333445555
Alice 666884444	Alice 666884444	Alice 666884444	Headquarters	Alice 666884444	ProjectZ	Alice 666884444
Jennifer 453453453	Jennifer 453453453	Jennifer 453453453	Headquarters	Jennifer 453453453	Computerization	Jennifer 453453453
Flanesh 888665555	Flanesh 888665555	Flanesh 888665555	Headquarters	Flanesh 888665555	Reorganization	Flanesh 888665555
Joyce 453453453	Joyce 453453453	Joyce 453453453	Headquarters	Joyce 453453453	Newsoftware	Joyce 453453453
Alfred 333445555	Alfred 333445555	Alfred 333445555	Headquarters	Alfred 333445555	Headquarters	Alfred 333445555
James 888665555	James 888665555	James 888665555	Headquarters	James 888665555	Headquarters	James 888665555

## Example of the result of a UNION operation

- UNION Example

Figure 6.3  
Result of the UNION operation  
RESULT ← RESULT1 ∪ RESULT2.

RESULT1	RESULT2	RESULT
Ssn	Ssn	Ssn
123456789	333445555	123456789
333445555	888665555	333445555
666884444		666884444
453453453		453453453
		888665555

## INTERSECTION

- INTERSECTION is denoted by ∩
- The result of the operation R ∩ S, is a relation that includes all tuples that are in both R and S
  - The attribute names in the result will be the same as the attribute names in R
- The two operand relations R and S must be "type compatible"

## SET DIFFERENCE

- SET DIFFERENCE (also called MINUS or EXCEPT) is denoted by -
- The result of  $R - S$ , is a relation that includes all tuples that are in R but not in S
  - The attribute names in the result will be the same as the attribute names in R
- The two operand relations R and S must be "type compatible"

## Example to illustrate the result of UNION, INTERSECT, and DIFFERENCE

(a) STUDENT		INSTRUCTOR		(b)	
Fn	Ln	Fname	Lname	Fn	Ln
Susan	Yao	John	Smith	Susan	Yao
Ramesh	Shah	Ricardo	Browne	Ramesh	Shah
Johnny	Kohler	Susan	Yao	Johnny	Kohler
Barbara	Jones	Francis	Johnson	Barbara	Jones
Amy	Ford	Ramesh	Shah	Amy	Ford
Jimmy	Wang			Jimmy	Wang
Ernest	Gilbert			Ernest	Gilbert
				John	Smith
				Ricardo	Browne
				Francis	Johnson

(c)		(d)		(e)	
Fn	Ln	Fn	Ln	Fname	Lname
Susan	Yao	Johnny	Kohler	John	Smith
Ramesh	Shah	Barbara	Jones	Ricardo	Browne
		Amy	Ford	Francis	Johnson
		Jimmy	Wang		
		Ernest	Gilbert		

**Figure 6.4**  
The set operations UNION, INTERSECTION, and MINUS. (a) Two union-compatible relations. (b)  $\text{STUDENT} \cup \text{INSTRUCTOR}$ . (c)  $\text{STUDENT} \cap \text{INSTRUCTOR}$ . (d)  $\text{STUDENT} - \text{INSTRUCTOR}$ . (e)  $\text{INSTRUCTOR} - \text{STUDENT}$ .

## Some properties of UNION, INTERSECT, and DIFFERENCE

- Notice that both union and intersection are *commutative* operations; that is
  - $R \cup S = S \cup R$ , and  $R \cap S = S \cap R$

## Some properties of UNION, INTERSECT, and DIFFERENCE

- Both union and intersection are *associative* operations; that is
  - $R \cup (S \cap T) = (R \cup S) \cap T$
  - $(R \cap S) \cap T = R \cap (S \cap T)$
- The minus operation is not commutative; that is, in general
  - $R - S \neq S - R$

## CARTESIAN PRODUCT

- CARTESIAN PRODUCT Operation
  - This operation is used to combine tuples from two relations in a combinatorial fashion.
  - Denoted by  $R(A_1, A_2, \dots, A_n) \times S(B_1, B_2, \dots, B_m)$
  - Result is a relation Q with degree  $n + m$  attributes:
    - $Q(A_1, A_2, \dots, A_n, B_1, B_2, \dots, B_m)$ , in that order.

## CARTESIAN PRODUCT

- The resulting relation state has one tuple for each combination of tuples—one from R and one from S.
- Hence, if R has  $n_R$  tuples (denoted as  $|R| = n_R$ ), and S has  $n_S$  tuples, then  $R \times S$  will have  $n_R * n_S$  tuples.
- The two operands do NOT have to be "type compatible"

## CARTESIAN PRODUCT

- Generally, CROSS PRODUCT is not a meaningful operation
  - Can become meaningful when followed by other operations
- Example (not meaningful):
  - FEMALE\_EMPS  $\leftarrow \sigma_{SEX='F'}(EMPLOYEE)$
  - EMP\_NAMES  $\leftarrow \pi_{FNAME, LNAME, SSN}(FEMALE_EMPS)$
  - EMP\_DEPENDENTS  $\leftarrow EMP_NAMES \times DEPENDENT$




FIGURE 8.4  
One possible database state for the COMPANY relational database schema

EMPLOYEE	DEPARTMENT	WORKS_ON	PROJECT	DEPENDENT
John B Smith 123456789 1965-01-09 731 Fondren, Houston, TX M 30000 333445555 5	Research 5 333445555 1988-05-22	123456789 1 32.5	ProductX 1 Boffere 5	333445555 Alice F 1966-04-05 Daughter
Franklin T Wong 933445555 1955-12-08 638 Voss, Houston, TX M 40000 888665555 5	Administration 4 987654321 1995-01-01	123456789 2 7.5	ProductY 2 Sugarland 5	333445555 Theodore M 1963-10-25 Son
Alice J Zelays 989897777 1968-01-18 3351 Castle, Spring, TX F 25000 987654321 4	Headquarters 1 888665555 1981-06-19	666884444 3 40.0	ProductZ 3 Houston 5	987654321 Joy F 1958-05-03 Spouse
Jennifer S Wallace 987654321 1941-06-20 381 Bony, Bellaire, TX F 43000 888665555 5		453403453 1 20.0	Computerization 10 Stafford 4	987654321 Abner M 1942-02-28 Spouse
Ramesh K Narayan 666884444 1982-09-15 975 Fire Oak, Humble, TX M 38000 333445555 5		453445555 2 20.0	Reorganization 20 Houston 1	123456789 Michael M 1988-01-04 Son
Joyce A English 453434563 1972-07-31 5631 Rice, Houston, TX F 25000 333445555 5		333445555 3 10.0	Newbenefits 30 Stafford 4	987654321 Alice F 1968-12-30 Daughter
Ahmad V Jabbar 987654321 1965-03-29 980 Dallas, Houston, TX M 25000 987654321 4		333445555 10 10.0		123456789 Elizabeth F 1967-05-05 Spouse
James E Borg 888665555 1937-11-10 450 Stone, Houston, TX M 55000 NULL 1		333445555 20 10.0		


## CARTESIAN PRODUCT example

Figure 8.5  
The CARTESIAN PRODUCT (CROSS PRODUCT) operation.

EMPLOYEE	DEPENDENT
Alice J Zelays 989897777 1968-01-18 3351 Castle, Spring, TX F 25000 987654321 4	333445555 Alice F 1966-04-05 Daughter
Franklin T Wong 933445555 1955-12-08 638 Voss, Houston, TX M 40000 888665555 5	333445555 Theodore M 1963-10-25 Son
Jennifer S Wallace 987654321 1941-06-20 381 Bony, Bellaire, TX F 43000 888665555 5	987654321 Joy F 1958-05-03 Spouse
Ramesh K Narayan 666884444 1982-09-15 975 Fire Oak, Humble, TX M 38000 333445555 5	987654321 Abner M 1942-02-28 Spouse
Joyce A English 453434563 1972-07-31 5631 Rice, Houston, TX F 25000 333445555 5	123456789 Michael M 1988-01-04 Son
Ahmad V Jabbar 987654321 1965-03-29 980 Dallas, Houston, TX M 25000 987654321 4	123456789 Alice F 1968-12-30 Daughter
James E Borg 888665555 1937-11-10 450 Stone, Houston, TX M 55000 NULL 1	123456789 Elizabeth F 1967-05-05 Spouse

## Example of applying CARTESIAN PRODUCT

- To keep only combinations where the DEPENDENT is related to the EMPLOYEE, we add a SELECT operation as follows
- Add:
 
$$ACTUAL\_DEPS \leftarrow \sigma_{SSN=ESSN}(EMP\_DEPENDENTS)$$
- Result:
 
$$RESULT \leftarrow \pi_{FNAME, LNAME, DEPENDENT\_NAME}(ACTUAL\_DEPS)$$




## Binary Relational Operations

- Join
- Division




## JOIN

- JOIN Operation (denoted by  $\bowtie$ )
  - The sequence of CARTESIAN PRODUCT followed by SELECT is used quite commonly to identify and select related tuples from two relations
  - This operation is very important for any relational database with more than a single relation, because it allows us combine related tuples from various relations




## JOIN

- The general form of a join operation on two relations  $R(A_1, A_2, \dots, A_n)$  and  $S(B_1, B_2, \dots, B_m)$  is:
 
$$R \bowtie_{\langle \text{join condition} \rangle} S$$
- where  $R$  and  $S$  can be any relations that result from general relational algebra expressions.



## JOIN

- Example: Suppose that we want to retrieve the name of the manager of each department.
- To get the manager's name, we need to combine each DEPARTMENT tuple with the EMPLOYEE tuple whose SSN value matches the MGRSSN value in the department tuple.
- $DEPT\_MGR \leftarrow DEPARTMENT \bowtie_{MGRSSN=SSN} EMPLOYEE$



One possible database state for the COMPANY relational database scheme.

EMPLOYEE									
Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1955-01-09	731 Fondren, Houston, TX	M	30000	333445555	5
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	M	40000	888665555	5
Alicia	J	Zelaya	989887777	1966-01-19	3321 Castle, Spring, TX	F	25000	987654321	4
Jennifer	S	Wallace	987654321	1941-06-20	291 Berry, Bellaire, TX	F	43000	888665555	4
Ramesh	K	Narayan	888665444	1962-09-15	975 Fire Oak, Humble, TX	M	38000	333445555	5
Joyce	A	English	453453453	1972-07-31	5831 Rice, Houston, TX	F	25000	333445555	5
Ahmad	V	Jabbar	987987987	1969-03-29	980 Dallas, Houston, TX	M	25000	987654321	4
James	E	Borg	888665555	1937-11-10	455 Stone, Houston, TX	M	55000	NULL	1

DEPARTMENT			
Dname	Dnumber	Mgr_ssn	Mgr_start_date
Research	5	333445555	1988-05-22
Administration	4	987654321	1995-01-01
Headquarters	1	888665555	1981-08-19

DEPT_LOCATIONS	
Dnumber	Location
1	Houston
4	Stafford
5	Bellaire
5	Sugarland
5	Houston

WORKS_ON		
Emp	Pno	Hours
123456789	1	32.5
123456789	2	7.5
888665444	3	40.0
453453453	1	20.0
453453453	2	20.0
333445555	2	10.0
333445555	3	10.0
333445555	10	10.0
333445555	20	10.0
989887777	30	30.0
989887777	10	10.0
987987987	10	35.0
987987987	30	5.0
987654321	30	20.0
987654321	20	15.0
888665555	20	NULL

PROJECT				
Pname	Pnumber	Plocation	Dnum	
ProductX	1	Bellaire	5	
ProductY	2	Sugarland	5	
ProductZ	3	Houston	5	
Computerization	10	Stafford	4	
Reorganization	20	Houston	1	
Newbenefits	30	Stafford	4	


DEPENDENT				
Esan	Dependent_name	Sex	Bdate	Relationship
333445555	Alice	F	1986-04-05	Daughter
333445555	Theodore	M	1983-10-25	Son
333445555	Joy	F	1958-05-03	Spouse
987654321	Abner	M	1942-02-28	Spouse
123456789	Michael	M	1988-01-04	Son
123456789	Alice	F	1985-12-30	Daughter
123456789	Elizabeth	F	1967-05-05	Spouse

## Example of applying the JOIN operation

DEPT_MGR									
Dname	Dnumber	Mgr_ssn	...	Fname	Minit	Lname	Ssn	...	
Research	5	333445555	...	Franklin	T	Wong	333445555	...	
Administration	4	987654321	...	Jennifer	S	Wallace	987654321	...	
Headquarters	1	888665555	...	James	E	Borg	888665555	...	


Figure 6.6  
Result of the JOIN operation

$DEPT\_MGR \leftarrow DEPARTMENT \bowtie_{MGRSSN=SSN} EMPLOYEE$




## JOIN

- The general case of JOIN operation is called a Theta-join:  $R \bowtie_{\theta} S$
- The join condition is called *theta*
- *Theta* can be any general boolean expression on the attributes of  $R$  and  $S$ ; for example:
  - $R.A_i < S.B_j$  AND  $(R.A_k = S.B_l$  OR  $R.A_p \times S.B_q)$



## EQUIJOIN


- The most common use of join involves join conditions with *equality comparisons* only
- Such a join, where the only comparison operator used is =, is called an EQUIJOIN
- The JOIN seen in the previous example was an EQUIJOIN






## NATURAL JOIN

- Another variation of JOIN called NATURAL JOIN – denoted by \*
- It was created to get rid of the second (superfluous) attribute in an EQUIJOIN condition.




## NATURAL JOIN

- Another example:  $Q \leftarrow R(A,B,C,D) * S(C,D,E)$
- The implicit join condition includes each pair of attributes with the same name, "AND"ed together:
  - $R.C=S.C$  AND  $R.D = S.D$
- Result keeps only one attribute of each such pair:
  - $Q(A,B,C,D,E)$




## NATURAL JOIN

- Example: To apply a natural join on the DNUMBER attributes of DEPARTMENT and DEPT\_LOCATIONS, it is sufficient to write:
  - $DEPT\_LOCS \leftarrow DEPARTMENT * DEPT\_LOCATIONS$



## NATURAL JOIN

- Only attribute with the same name is DNUMBER
- An implicit join condition is created based on this attribute:
  - $DEPARTMENT.DNUMBER=DEPT\_LOCATIONS.DNUMBER$



One possible database state for the COMPANY relational database schema. Figure 5.6

Empno	Empname	Deptname	Deptnumber	Mgr_ssn	Mgr_start_date
1	John Smith	Research	5	333445555	1988-05-22
2	Franklin T. Wong	Administration	4	987654321	1995-01-01
3	Alice J. Zelaya	Research	5	888665555	1988-05-22
4	Jennifer S. Wallace	Administration	4	987654321	1995-01-01
5	Ramesh K. Narayan	Research	5	888665555	1988-05-22
6	Joyce A. English	Administration	4	987654321	1995-01-01
7	Ahmad V. Jabbar	Research	5	888665555	1988-05-22
8	James E. Borg	Administration	4	987654321	1995-01-01

Deptname	Dnumber	Mgr_ssn	Mgr_start_date
Research	5	333445555	1988-05-22
Administration	4	987654321	1995-01-01
Headquarters	1	888665555	1981-06-19

Deptname	Dnumber	Plocation	Dnum
1	1	Houston	5
4	1	Stafford	4
5	1	Bellaire	5
5	2	Sugarland	5
5	3	Houston	5
5	4	Stafford	4
5	5	Houston	5
5	6	Sugarland	5
5	7	Houston	5
5	8	Stafford	4

Empno	Pno	Hours
1	32.5	
2	7.5	
3	40.0	
4	20.0	
5	20.0	
6	10.0	
7	10.0	
8	10.0	
9	10.0	
10	10.0	
11	10.0	
12	10.0	
13	10.0	
14	10.0	
15	10.0	
16	10.0	
17	10.0	
18	10.0	
19	10.0	
20	10.0	
21	10.0	
22	10.0	
23	10.0	
24	10.0	
25	10.0	
26	10.0	
27	10.0	
28	10.0	
29	10.0	
30	10.0	
31	10.0	
32	10.0	
33	10.0	
34	10.0	
35	10.0	
36	10.0	
37	10.0	
38	10.0	
39	10.0	
40	10.0	
41	10.0	
42	10.0	
43	10.0	
44	10.0	
45	10.0	
46	10.0	
47	10.0	
48	10.0	
49	10.0	
50	10.0	

Empno	Dependent_name	Sex	Bdate	Relationship
333445555	Alice	F	1986-04-05	Daughter
333445555	Theodore	M	1983-10-25	Son
333445555	Joy	F	1956-05-03	Spouse
987654321	Abner	M	1945-05-28	Spouse
123456789	Michael	M	1988-01-04	Son
123456789	Alice	F	1986-12-30	Daughter
123456789	Elizabeth	F	1957-05-05	Spouse

## NATURAL JOIN example


a)

Pname	Pnumber	Plocation	Dnum	Dname	Mgr_ssn	Mgr_start_date
ProductX	1	Bellaire	5	Research	333445555	1988-05-22
ProductY	2	Sugarland	5	Research	333445555	1988-05-22
ProductZ	3	Houston	5	Research	333445555	1988-05-22
Computerization	10	Stafford	4	Administration	987654321	1995-01-01
Reorganization	20	Houston	1	Headquarters	888665555	1981-06-19
Newbenefits	30	Stafford	4	Administration	987654321	1995-01-01

b)

Dname	Dnumber	Mgr_ssn	Mgr_start_date	Location
Headquarters	1	888665555	1981-06-19	Houston
Administration	4	987654321	1995-01-01	Stafford
Research	5	333445555	1988-05-22	Bellaire
Research	5	333445555	1988-05-22	Sugarland
Research	5	333445555	1988-05-22	Houston

Figure 6.7  
Results of two NATURAL JOIN operations.  
(a)  $PROJ\_DEPT \leftarrow PROJECT * DEPT$ ,  
(b)  $DEPT\_LOCS \leftarrow DEPARTMENT * DEPT\_LOCATIONS$ .



## Complete Set of Relational Operations

- The set of operations including SELECT  $\sigma$ , PROJECT  $\pi$ , UNION  $\cup$ , DIFFERENCE  $-$ , RENAME  $\rho$ , and CARTESIAN PRODUCT  $\times$  is called a *complete set* because any other relational algebra expression can be expressed by a combination of these five operations.
- For example:
  - $R \cap S = (R \cup S) - ((R - S) \cup (S - R))$
  - $R \bowtie_{\langle \text{join condition} \rangle} S = \sigma_{\langle \text{join condition} \rangle} (R \times S)$



## Aggregate Functions and Grouping

- Common functions applied to collections of numeric values include
  - SUM, AVERAGE, MAXIMUM, and MINIMUM.
- The COUNT function is used for counting tuples or values.



## Aggregate Functions and Grouping

- Use of the Aggregate Functional operation  $\exists$ 
  - $\exists_{\text{MAX Salary}}(\text{EMPLOYEE})$  retrieves the maximum salary value from the EMPLOYEE relation
  - $\exists_{\text{MIN Salary}}(\text{EMPLOYEE})$  retrieves the minimum Salary value from the EMPLOYEE relation



## Aggregate Functions and Grouping

- $\exists_{\text{SUM Salary}}(\text{EMPLOYEE})$  retrieves the sum of the Salary from the EMPLOYEE relation
- $\exists_{\text{COUNT SSN, AVERAGE Salary}}(\text{EMPLOYEE})$  computes the count (number) of employees and their average salary



## Aggregate Functions and Grouping

- Grouping can be combined with Aggregate Functions
- Example: For each department, retrieve the DNO, COUNT SSN, and AVERAGE SALARY



## Aggregate Functions and Grouping

- A variation of aggregate operation  $\exists$  allows this:
  - Grouping attribute placed to left of symbol
  - Aggregate functions to right of symbol
  - $\text{DNO } \exists_{\text{COUNT SSN, AVERAGE Salary}}(\text{EMPLOYEE})$



