DIT250 / BIT180 - MATHEMATICS

- ASSESSMENT
  - CA – 40%
    - 3 TESTS
  - EXAM – 60%
- LECTURE SLIDES
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  - Lecture notes
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Number Bases

- In this lesson we shall discuss different Number Bases, specifically those used by the computer
- These include:
  - decimal numbers (base ten)
  - binary numbers (base two)
  - octal numbers (base eight)
  - Hexadecimal numbers (base sixteen)

Decimal numbers (base ten),

- Numbers used by humans to quantify items
- It's called base ten because…?
- Symbols used – 0,1,2,3,4,5,6,7,8 and 9
- To count in base ten, you go from 0 to 9, then do combinations of two digits starting with 10 all the way to 99

Decimal numbers (base ten),

- After 99 comes three-digit combinations from 100 – 999, etc.
- This combination system is true for any base you use.
- The only difference is how many digits you have before you go to the next combination

Decimal numbers (base ten),

- Have place values of powers of ten
  - Eg 1
  - 246_{10}
    - 2 place value - 10^2
    - 4 place value – 10^1
    - 6 place value – 10^0

Decimal numbers (base ten),

- Eg 2
- 46.57_{10}
  - 4 place value - 10^1
  - 6 place value – 10^0
  - 5 place value – 10^{-1}
  - 5 place value – 10^{-2}

Binary numbers (base two)

- Numbers used and understood by computers
- Symbols used 0 and 1
- To count in base two,
  - you count 0,1, then switch to two digit combinations, 10,11, then to three digit combos, 100, 101,110,111, then four digit, 1000, ______,________, …, 1111

Binary numbers (base two)

- Have place values of powers of two
  - Eg 1
  - 110_2
    - 1 place value - 2^2
    - 1 place value – 2^1
    - 0 place value – 2^0

Binary numbers (base two)

- Eg 2
- 11.10_{10}
  - 1 place value - 2^1
  - 1 place value – 2^0
  - 1 place value – 2^{-1}
  - 0 place value – 2^{-2}

Octal numbers (base eight),

- Numbers used by machine language programmers as short hand for binary numbers
- Three binary digits are equivalent to 1 octal digit
  - Eg $6_8 \approx 110_2$
- Symbols used – 0 , 1, 2, 3, 4 ,5, 6 and 7

Octal numbers (base eight),

- Here is the base eight counting sequence
  - 0,1,2,3,4,5,6,7,10,11,12,13,…77
  - 100,101,102,103,104,105,106,107
  - 110,111, etc.

Octal numbers (base eight),

- Have place values of powers of eight
  - Eg 1
  - 456_8
    - 4 place value - 8^2
    - 5 place value – 8^1
    - 6 place value – 8^0

Octal numbers (base eight),

- Eg 2
- 34.56_8
  - 3 place value - 8^1
  - 4 place value – 8^0
  - 5 place value – 8^{-1}
  - 6 place value – 8^{-2}

Hexadecimal numbers (base sixteen)

- Numbers used by machine and assembly language programmers to help simply low level programming
- Four binary digits are equivalent to 1 octal digit
  - Eg $9_{16} \approx 1001_2$
- Symbols used – 0 , 1, 2, 3, 4 ,5, 6,7,8,9,10, 11, 12, 13,14 and 15
- Symbols 10, 11, 12, 13, 14 and 15 replaced by letters A, B, C, D, E and F respectively

Hexadecimal numbers (base sixteen)

- Here's the single digit sequence for base sixteen:

  0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F
- Then the two-digit combos:

  10,11,12,…19,1A,1B,1C,1D,1E,1F,20, 21,22,…2D,2E,2F,30,31,…FF

Hexadecimal numbers (base sixteen)

- Have place values of powers of sixteen
  - Eg 1
  - $A79_{16}$
    - A place value - 16^2
    - 7 place value – 16^1
    - 9 place value – 16^0

Hexadecimal numbers (base sixteen)

- Eg 2
- $E6.A8_{16}$
  - E place value - 16^1
  - 6 place value – 16^0
  - A place value – 16^{-1}
  - 8 place value – 16^{-2}

Base conversion

- To convert from base ten to another base, such as base two, eight, or sixteen, is an important skill for computer scientists and programmers.
- The next section shows how to do this.

Base Ten to Base Two

- Let's take the value 27 and convert it into base 2.
- Here's the process:
  - Divide 27 by 2
  - The answer is 13, remainder 1
  - Divide 13 by 2
- Answer is 6, remainder 1

Base Ten to Base Two

- Continue until the answer is 1.
  - 6 divided by 2 = 3, remainder 0
  - 3 divided by 2 = 1, remainder 1

Base Ten to Base Two

- Now take the last answer, 1, and all of the remainders in reverse order, and put them together…11011
- 27 base 10 = 11011 base two

Base Ten to Base Two

- Here's an easy way to do it on paper

```
2 | 27   1
    13
```

- 27 divided by 2 = 13, R 1

Base Ten to Base Two

```
2 | 27   1
2 | 13   1
     6
```

- 13 / 2 = 6, R 1

Base Ten to Base Two

```
2 | 27   1
2 | 13   1
2 |  6   0
     3
```

- 6 / 2 = 3, R 0

Base Ten to Base Two

2	27	1
2	13	1
2	6	0
2	3	1
	1	

- 3 / 2
  = 1, R 1

Base Ten to Base Two

2	27	1
2	13	1
2	6	0
2	3	1
	1	

- Stop, and write the answer

Base Ten to Base Two

2	27	1
2	13	1
2	6	0
2	3	1
	1	

11011

Base 2 to base 10

- Use place values to convert.
- Eg1. Convert 11011_2 to Base 10

1	1	0	1	1
2^4	2^3	2^2	2^1	2^0
16x1	8x1	4x0	2x1	1x1
16+	8+	0+	2+	1
				$=27_{10}$

Base 2 to base 10

- Eg2. Convert 1111.101_2 to Base 10

1	1	1	1	.	1	0	1
2^3	2^2	2^1	2^0	.	2^{-1}	2^{-2}	2^{-3}
8x1	4x1	2x1	1x1	.	1/2	0	1/8
8+	4+	2+	1+		½+	0+	1/8

=15.625

Base Ten to Base Eight

- Let's again take the value 27 and convert it into base 8.
- Same process:
  - Divide 27 by 8
  - The answer is 3, remainder 3
  - Stop! You can't divide anymore because the answer is less than 8

Base Ten to Base Eight

- The last answer was 3, and the only remainder was 3, so the base eight value is 33, base 8.

Base Ten to Base Eight

- Use the same method on paper

8 | 27 3

- 27 divided by 8 = 3, R 3
- 27, base 10 = 33, base 8

Base 8 to Base 10

- Use place values to convert
- Eg 1 Covert 2657_8 to Base 10

2	6	5	7
8^3	8^2	8^1	8^0
512x2	64x6	8x5	1x8
1024+	384+	40+	8+

1456_{10}

Base 8 to Base 10

- Eg2 Covert 327.24_8 to Base 10

3	2	7	.	2	4
8^2	8^1	8^0		8^{-1}	8^{-2}
64x3	8x2	1x7		1/8x2	1/64 x 4
192+	16+	7+		¼+	1/16

=215+5/16
=215.3125

Exercises

- Now try the same values for base eight.
  6. 16_{10} = __________$_8$
  7. 47_{10} = __________$_8$
  8. 145_{10} = __________$_8$
  9. 31_{10} = __________$_8$
  10. 32_{10} = __________$_8$

Base Ten to Base Sixteen

- Finally we'll convert 27 into base 16.
  - Divide 27 by 16
  - The answer is 1, remainder 11
  - Stop! You can't divide anymore because the answer is less than 16

Base Ten to Base Sixteen

- The last answer was 1, and the only remainder was 11, which in base 16 is the letter B, so the base sixteen value is 1B, base 16.

Base Ten to Base Sixteen

- Again, the same method on paper

16 | 27 11 (B)

- 27 divided by 16 = 1, R 11 or B
- 27, base 10 = 1B, base 16

Base 16 to Base 10

- E.g Covert $12AE_{16}$ to base 10

1	2	A	E
16^3	16^2	16^1	16^0
4096x1	256x2	16x10	1x 15
4096+	512+	160+	15

$=4783_{10}$

- EG 2, Convert $62A.48_{16}$ TO BASE 10

6	2	A	.	4	8
16^2	16^1	16^0		16^{-1}	16^{-2}
256x6	16x2	1x10		1/16x4	1/256 x 4
1536+	32+	10+		¼+	1/32

=1578+9/32
=1578.2812

Convert from Base 2 to Base 8

- Using the fact that 3 binary digits are equivalent to one octal digit.
- Eg1. Convert 1001110011
- Group the bits in 3s beginning with the least significant bit
- 001 001 110 011
- Convert the individual groups to base 10.
  - Ie 001 = 1
    001 = 1
    110 = 6
    011 = 3

Therefore 1001110011_2 equivalent to 1163_{10}

Convert from Base 2 to Base 8

- Eg2. Convert 1110011.01101_2
- Group the bits in 3s beginning from the decimal point
- $001\ 110\ 011.011\ 010_2$
- Ie 001 = 1
  - 110 = 6
  - 011 =3
  - 011=3
  - 010 =2
- Therefore $1110011.01101_2 = 163.32_{10}$

Convert from Base 8 to Base 2

- Using the similar fact that 3 binary digits are equivalent to one octal digit and convert individual digits to base 2 and form groups of 3.
- Eg 1 convert 6752_8 to base 2
- 6 = 110
- 7 = 111
- 5 = 101
- 2 = 010
- Therefore $6752_8 = 110111101010_2$

Convert from Base 8 to Base 2

- Eg. 2, Convert 435.465_8 to base 2
- 4 = 100
- 3 = 011
- 5 = 101
- 4=100
- 6 = 110
- 5 =101
- Therefore $435.465_8 = 100011101.100110101_2$

Convert from Base 2 to Base 16

- Using the fact that 4 binary digits are equivalent to one hex digit.
- Eg1. Convert 100111001111
- Group the bits in 4s beginning with the least significant bit.
- 1001 1100 1111
- 1001 = 9
- 1100 = 12 = C
- 1111 = 15 =F
- Therefore = $100111001111_2 = 9CF_{16}$

Convert from Base 2 to Base 16

- Eg2. Convert 111001111.01110101_2
- Group the bits in 4s beginning from the decimal point
- 111001111.01110101_2
- 0001 1100 1111. 0111 0101
- 0001 = 1
- 1100 = 12 = C
- 1111 = 15 = F
- 0111 = 7
- 0101 = 5
- Therefore $111001111.01110101_2 = 1CF.75_{16}$