

Number systems arithmetic

- In this lesson we shall discuss different arithmetic operations on number bases
- These include:
 - Addition
 - Subtraction
 - Multiplication and
 - division

Binary arithmetic

- Addition
 - Eg 1
 - Add 100010 and 11101

1	0	0	0	1	0
+	1	1	1	0	1
1	1	1	1	1	1

- Eg 2
- Add 100010 and 11101

1	0	1	0	1	1
+		1	1	0	1
1	1	1	0	0	0

Subtraction

- Three methods
 - Direct method
 - 1st Compliment method
 - 2nd Compliment method

Direct method

- Eg. 1
- Subtract 10001 from 111011

1	1	1	0	1	1
-	1	0	0	0	1
1	0	1	0	1	0

Direct method

- Eg. 2
- Subtract 11111 from 111101

1	1	1	1	0	1
-	1	1	1	1	1
1	1	1	1	1	0

Direct method

- Eg. 3
- Subtract 10111 from 110001

1	1	0	0	0	0	1
-	1	0	1	1	1	1
1	1	0	1	0		

1st Compliment

- To find the 1st compliment of a given binary number
 - Invert the individual digits of a given number,
 - Ie . What was a zero becomes 1 and a 1 becomes a zero.
 - Eg the 1st compliment of 1001110 is 0110001

1st Compliment method

- Eg.
- Subtract 10001 from 111011

1	1	1	0	1	1
-	1	0	0	0	1

Step 1

- Equalise the number of digits for the given numbers by adding necessary zeros.
- Ie

1	1	1	0	1	1
-	0	1	0	0	1

Step 2

- Change the number below to its first compliment, ie

1	1	1	0	1	1
-	1	0	1	1	0

Step 3

- Add the two numbers

1	1	1	0	1	1
+	1	0	1	1	0
1	1	0	1	0	1

■ Step 4

■ Add the leading one to the remaining digits

$$\begin{array}{r}
 - \quad 1 \quad 0 \quad 1 \quad 1 \quad 1 \quad 0 \\
 \boxed{1} \quad 1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 1 \\
 \hline
 \\
 + 1
 \end{array}$$

■ Final answer

$$\begin{array}{r}
 1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 1 \\
 + \\
 \hline
 \boxed{1 \quad 0 \quad 1 \quad 0 \quad 1 \quad 0}
 \end{array}$$

Final Answer

2nd Complement

- To find the 2st complement of a given binary number
 - First, find the first complement of the given number,
 - Then add a 1 to the first complement number
 - Eg the 2st complement of 1001110 is.
 - 1st complement = 0110001
 - 2nd Complement = 0110001 + 1
 - = 0110010

2nd Complement subtraction

■ Eg.

- Subtract 10111 from 110001

$$\begin{array}{r}
 1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 1 \\
 - \quad 1 \quad 0 \quad 1 \quad 1 \quad 1 \\
 \hline
 \\

 \end{array}$$

2nd Complement subtraction

■ Step 2

- Equalise the number of digits for the given numbers by adding necessary zeros.
- Ie,

$$\begin{array}{r}
 1 \quad 1 \quad 1 \quad 0 \quad 1 \quad 1 \\
 - \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 1 \\
 \hline
 \\

 \end{array}$$

2nd Complement subtraction

■ Step 2

- Change the number below to its first complement, ie

$$\begin{array}{r}
 1 \quad 1 \quad 1 \quad 0 \quad 1 \quad 1 \\
 - \quad 1 \quad 0 \quad 1 \quad 1 \quad 1 \quad 0 \quad 1^{1st C} \\
 \hline
 \\

 \end{array}$$

2nd Complement subtraction

- Step 3
 - Change the number below to its second complement, ie

	1	1	1	0	1	1
-	1	0	1	1	1	1 2 st C

2nd Complement subtraction

- Step 3
 - Add the two numbers

	1	1	1	0	1	1
+	1	0	1	1	1	1 2 st C
1	1	0	1	0	1	0

2nd Complement subtraction

- Step 5
 - Delete the leading 1

	1	1	1	0	1	1
+	1	0	1	1	1	1 2 st C
1	1	0	1	0	1	0

Delete

2nd Complement subtraction

- Final answer

	1	1	1	0	1	1
+	1	0	1	1	1	1 2 st C
1	0	1	0	1	0	

Binary multiplication

- Multiply 1011 by 11

		1	0	1	1
x				1	1
	1	0	1	1	0
+		1	0	1	1
1	0	0	0	0	1

Binary division

- Divide 110 into 1110111

		10011	r	101
110)	1110111		
		- 110		
			1011	
			- 110	
				1011
				- 110
				101

Octal arithmetic

- Addition
 - Eg 1
 - Add 5342_8 and 423_8

5	3	4	2
+	4	2	3
5	7	6	5

■ Addition

- Eg 2
- Add 6354_8 and 545_8

6	3	5	4
+	5	4	5
7	1	2	1

Subtraction

- Two methods
 - Direct method
 - 7th Compliment method

Direct method

- Eg 1
- Subtract 543_8 from 6765_8

6	7	6	5
-	5	4	3
6	2	2	2

- Eg 2
- Subtract 543_8 from 6765_8

5	3	4	2
-	7	5	3
4	3	6	7

7th Compliment

- To find the 7th compliment of a given octal number
 - Subtract individual digits of a given octal number from 7,
 - Eg the 7th compliment of 4563_8 is 3214_8

7th Compliment method

- Eg 1
 - Subtract 543_8 from 6765_8
- Step 1
 - Equalise the number of digits for the given numbers by adding necessary zeros.

	6	7	6	5
-	0	5	4	3

- Step 2
 - Change the number below to its 7th compliment, ie

	6	7	6	5
	7	2	3	4

- Step 3
 - Add the two numbers

	6	7	6	5
+	7	2	3	4
	1	6	2	2
				1

- Step 4
 - Add the leading 1 to the rest of the digits in the resultant sum

	6	2	2	1
+				1
	6	2	2	2

Octal Multiplication

- Multiply $345_8 \times 12_8$

	3	4	5
X		1	2
	3	4	5
	7	1	2
	4	3	6
			2

Octal division

- Convert the given numbers to binary and divide in binary and convert the final answer back to octal.

Hex arithmetic

- Addition
 - Eg 2
 - Add $A39B_{16}$ and $BA8_{16}$

A	3	9	B
+	B	A	8
A	F	4	3

Subtraction

- Two methods
 - Direct method
 - 15th Compliment method

Direct method

- Eg 1
 - Subtract $A9C_{16}$ from $1B62_{16}$

1	B	6	2
-	A	9	C
1	0	C	6

15th Compliment

- To find the 15th compliment of a given HEX number
 - Subtract individual digits of a given Hex number from 15,
 - Eg the 15th compliment of $A5E3_{16}$ is $5A1C_{16}$

15th Compliment method

- Eg 1
 - Subtract $A9C_{16}$ from $1B62_{16}$
- Step 1
 - Equalise the number of digits for the given numbers by adding necessary zeros.

1	B	6	2
-	0	A	9

- Step 2
 - Change the number below to its 15th compliment, ie

1	B	6	2
F	5	6	3

- Step 3

- Add the two numbers

	1	B	6	2
+	F	5	6	3
	1	1	0	C

- Step 4

- Add the leading 1 to the rest of the digits in the resultant sum

	1	0	C	5
+				1
	1	0	C	6

Hex Multiplication

- Multiply $A25_{16} \times 12_{16}$

	A	2	5
	X	1	2
A	2	5	0
1	4	4	A
	B	6	9

Hex division

- Convert the given numbers to binary and divide in binary and convert the final answer back to Hex.