Parallel Algorithm Construction

- Parallel algorithms for MIMD machines can be divided into 3 categories,
- these are :
  - Pipelined Algorithms / Algorithmic Parallelism
  - Partitioned Algorithms / Geometric Parallelism
  - Asynchronous / Relaxed Algorithms

Pipelined Algorithms / Algorithmic Parallelism

- A pipelined algorithm is an ordered set of (possibly different) processes in which the output of each process is the input to its successor.
- The input to the first process is the input to the algorithm
- The output from the last process is the output of the algorithm.

Pipelined Algorithms

- Typically each processor forms part of a pipeline and
- performs only a **small part of the algorithm.**
- Data then flows through the system (pipeline) being operated on by each processor in succession.

Example

- Say it takes 3 steps A, B & C to assemble a widget and assume each step takes one unit of time
- **Sequential widget assembly machine:**
  - Spends 1 unit of time doing step A followed by 1 unit of time doing step B, followed by 1 unit of time doing step C
  - So a sequential widget assembler produces 1 widget in 3 time units, 2 in 6 time units etc.
    i.e. one widget every 3 units

Example

- **Pipelined widget assembly machine**
  - Say we use a 3 segment pipeline where each of the subtasks (A, B or C) is assigned to a segment
  - i.e. the machine is split into 3 smaller machines; one to do step A, one for step B and one for step C and which can operate simultaneously.

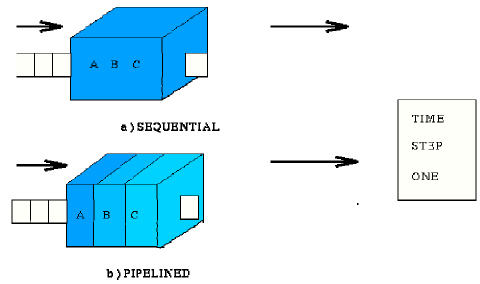
Example

- The first machine performs step A on a new widget **every time step** and
- passes the partially assembled widget to the second machine which performs step B.
- This is then passed onto the third machine to perform step C

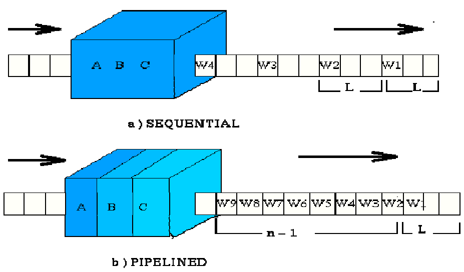
Example

- This produces the first widget in 3 time units (as the sequential machine),
- but after this initial **startup time** one widget appears every time step.
- i.e. the second widget appears at time 4
  the third widget appears at time 5 etc.

Example



So the final result looks like this



Pipelined Algorithms

- In general
  - if L is the number of steps to be performed
  - and T is the time for each step
  - and n is the number of items (widgets)
  - then Time Sequential = LTn
  - and Time Parallel = [L + n-1]T

Pipelined Algorithms

- T = 1, L = 100, n = 10^6
- then Tseq = 10^8 and Tpipe = 100 + 10^6 - 1 = 10^6 + 99
- Speedup = Tseq / Tpipe = 10^8 / (10^6 +99) = 100
- i.e. 100 fold increase in speed.
- In general as n tends to infinity speedup tends to L.

Geometric Parallelism / Partitioned Algorithms

- These algorithms arise when there is a natural way to decompose **the data set** into smaller "chunks" of data,
- which are then allocated to individual processors.
- Thus each processor contains more or less the same code but operates on a subset of the total data.
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Partitioned Algorithms

- The solution to these subproblems are then combined to form the complete solution.
- Depending on the algorithm being solved this combining of solutions usually implies
- communication synchronization among the processors.
- Synchronization means constraining a particular ordering of events.

Example

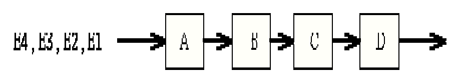
- if data needs to be communicated between processors after each iteration of a numerical calculation then this implies synchronization between processes.
- Thus partitioned algorithms are sometimes called **synchronous algorithms**

Partitioned Algorithms

- To illustrate the difference between pipelined and partitioned algorithms consider the following:
  - Say an algotithm consists of 4 parts A, B, C and D and
  - this algorithm is to operate on a data set E consisting of 4 subsets E1, E2 , E3 and E4
  - (e.g. divide up matrix into submatrix)

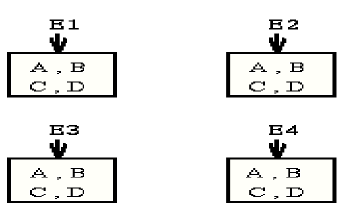
Partitioned Algorithms

- The pipelined algorithm would consist of 4 processors performing A, B, C, or D.
- The complete data set would then pass through all 4 processors.



Partitioned Algorithms

- However in the partitioned algorithm the four processors all perform A, B, C and D but only on a subset of the data

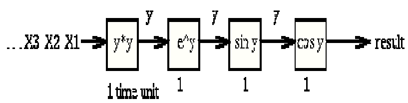


Partitioned Algorithms

- i.e. In pipelined algorithms the **algorithm** is distributed among the processors whereas in partitioned algorithms the **data** is distributed among the processors.

Example

- Say we want to calculate Fi = cos(sin e^sqr(xi)) for x1, x2 ,....x6 using 4 processors.
- **Pipelined Version**

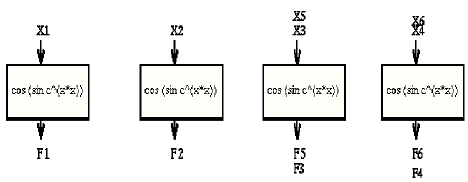


Example

- F1 is produced in 4 time units
  F2 is produced at time 5
  i.e. time = 4 + (6-1) = 9 units
  ==> SPEEDUP = 24 / 9 = 2.6

Example

- **Partitioned Version**
- This time each processor performs the complete algorithm i.e. cos(sin e^sqr(x)) but on its own data.



Example

- i.e. time = 8 units
  ==> SPEEDUP = 24 / 8 = 3
  ==> EFFICIENCY = 75%
- Efficiency is calculated by dividing speedup by number of processors
- E=S/n

Asynchronous / Relaxed Parallelism

- In relaxed algotithms there is no explicit dependency between processes,
- as occurs in synchronized algorithms.
- Instead relaxed algorithms **never** wait for input.
- If they are ready they use the **most recently available data**

Relaxed Parallelism

- To illustrate this consider the following.
- Say we have two processors A and B. A produces a sequence of numbers a1, a2 ..
- B inputs ai and performs some calculation F which uses ai.
- Say that B runs much faster than A.

Example

- **Synchronous Operation**
- A produces a1 passes it to B which calculates F1;
- A produces a2 passes it to B which calculates F2;
- i.e. B waits for A to finish (since B is faster than A) etc..

Example

- **Asynchronous Operation**
- A produces a1 passes it to B which calculates F1
- but now A is still in the process of computing a2
- so instead of waiting B carries on and calculates F2 (based on old data i.e. a1 and therefore may not be the same as F2 above)and
- continues to calculate F using the old data until a new input arrives
- e.g. Fnew = Fold + ai

Relaxed Parallelism

- The idea in using asynchronous algorithms is that all processors are kept busy and never remain idle (unlike synchronous algorithms) so speedup is maximized.
- A drawback is that they are difficult to analyse (because we do not know what data is being used) and
- also an algorithm that is known to work (e.g. converge) in synchronous mode may not work (e.g diverge) in asynchronous mode.

Relaxed Parallelism

- Consider the Newton Raphson iteration for solving
- F (x) = 0
- where F is some non-linear function
- i.e. Xn+1 = Xn - F(Xn)/F'(Xn)......(1)
  generates a sequence of approximations to the root, starting from a value X0.

Relaxed Parallelism

- Say we have 3 processors
- P1 : given x, P1 calculates F (x) in time t1, units and sends it to P3
- P2 :given y, P2 calculates F'(y) in time t2 units and sends it to P3
- P3 : given a, b, c, P3 calculates d = a - b/c in time t3 units;
- if | d-a | > Epsilon then d is sent to P1 and P2 otherwise d is output.

Example

- Serial Mode
  - P1 computes F(Xn)
  - then P2 computes F'(Xn)
  - then P3 computes Xn+1 using (1)
  - So time per iteration is t1 + t2 + t3
  - If k iterations are necessary for convergence then total time is k (t1 + t2 + t3)

Example

- **Synchronous Parallel Mode.**
  - P1 and P2 compute F(Xn) and F'(Xn) simultaneously and
  - when BOTH have finished the values F(Xn) and F'(Xn) are used by P3 to compute Xn+1
  - Time per iteration is max(t1, t2) + t3
- Again k iterations will be necessary so total time is **k [max(t1, t2) + t3]**
  X1 = X0 - F(X0)/F'(X0) ...etc

Relaxed Parallelism

- **Asynchronous Parallel Mode**
  - P1 and P2 begin computing as soon as a new input value is made available by P3 and they are ready to receive it,
  - P3 computes a new value using (1) as soon as EITHER P1 OR P2 provide a new input
  - i.e. (1) is now of the form
- **Xn+1 = Xn - F(SXi)/F'(Xj)**